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DSP Demodulation

by

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ABSTRACT

With high-speed digital signal processing (DSP) and signal digitization hardware becoming available at a reasonable cost, it is now practical to propose DSP demodulation for future designs. This paper presents the necessary theory and governing equations for practical DSP demodulators that can be implemented for a wide variety of applications. The Generalized Costas Demodulator (GCD) will be emphasized because it is capable of recovering the demodulation functions of an arbitrary bandlimited signal.

1. INTRODUCTION

The use of digital signal processing for modem designs has been very popular the past few years. The effective application of DSP to these designs requires a good understanding of the governing equations associated with modulation/demodulation and the signal digitization process. In particular, a good understanding of quadrature detectors, aliasing, decimation, and the process of carrier recovery is required. This paper has been written to contribute to this understanding by presenting the necessary governing equations.

A good DSP design depends on a good signal digitization technique. The designer must be concerned with the linearity, dynamic range, and noise performance of the digitizer. It is especially difficult to achieve the dynamic range needed for most receiver designs because of 1) digitizer range limitations and 2) quantization noise produced during digitization. Usually, automatic gain control (AGC) or hard limiters are used to reduce the dynamic range requirements for the digitizer. Recently [1] [2] sigma-delta modulation has been proposed for use in signal digitizers as a method of achieving high dynamic range and good linearity.

The governing equations for modulation/demodulation will now be presented. It will be assumed that adequate signal digitization has been achieved. An illustrative modulation/demodulation example is included in the appendix.

2. CONCEPTUAL SIGNAL MODEL - TRANSMITTER

To adequately describe DSP demodulation, it is first necessary to develop precise conceptual models for the transmitted and received signals. The fundamental model underpinning all analysis is that of a noiseless narrowband signal that can be analytically represented in:

1. amplitude/phase form,

$$s(t) = r(t) A_c \cos[2\pi f_c t + \phi_c + \phi(t)] \quad (1)$$

2. or quadrature form [3],

$$s(t) = a(t) A_c \cos[2\pi f_c t + \phi_c] - b(t) A_c \sin[2\pi f_c t + \phi_c] \quad (2)$$

3. or upper/lower sideband form [4]:

$$s(t) = [u(t) + l(t) + k_c] A_c \cos[2\pi f_c t + \phi_c] - [\tilde{u}(t) - \tilde{l}(t)] A_c \sin[2\pi f_c t + \phi_c] \quad (3)$$

The signal carrier is modeled as a cosine with amplitude A_c, frequency f_c, and constant phase, ϕ_c. The constant phase will be used to model the effect of an arbitrary reference time at the receiver. The effect of modulation on the carrier is modeled using the envelope and phase functions, $r(t)$ and $\phi(t)$, the quadrature functions, $a(t)$ and $b(t)$, or the upper and lower sideband functions, $u(t)$ and $l(t)$. These modulation functions all represent some coded form of an analog or digital information source. For this narrowband model, the modulation of the carrier must be restricted such that the spectrum of $s(t)$ is zero outside the frequency range f_1 to f_2 where $(f_2 - f_1) << f_c$ and all the transmitted spectrum is passed by the preselector and IF filters in the receiver. An unmodulated carrier will be represented by the constants $r(t) = 1$ and $\phi(t) = 0$.

Using trigonometric identities [5], the modulation functions can be related as follows:

$$a(t) = r(t)\cos[\phi(t)] \qquad b(t) = r(t)\sin[\phi(t)] \quad (4)$$

$$r(t) = \sqrt{a^2(t) + b^2(t)} \qquad \phi(t) = \arctan\left[\frac{b(t)}{a(t)}\right] \quad (5)$$

$$a(t) = u(t) + l(t) + k_c \qquad b(t) = \tilde{u}(t) - \tilde{l}(t) \tag{6}$$

$$u(t) = \frac{1}{2}[a(t) - \tilde{b}(t)] \qquad l(t) = \frac{1}{2}[a(t) + \tilde{b}(t)] \tag{7}$$

The modulation functions may assume any real value except for the envelope function, $r(t)$, which must always be non-negative.

The amplitude spectrum of $s(t)$, if it exists, can be obtained by applying the Fourier transform to the quadrature form of the signal. Because $s(t)$ is a narrowband signal, the spectrum is nonzero only near the frequencies $\pm f_c$. Furthermore, a sinusoidal carrier frequency component is present only if $A(f \pm f_c)$ or $B(f \pm f_c)$ contains a Dirac delta function [3]. The tilde (~) denotes the Hilbert transform needed to generate SSB signals. The constant, k_c, represents the magnitude of any carrier that is transmitted with the two sidebands.

3. CONCEPTUAL SIGNAL MODEL - RECEIVER

The simplified signal model at the receiver will include the effects of additive channel noise and carrier phase shift due to propagation delay. Doppler, dispersion, slow carrier amplitude variations, and multipath effects will not be modeled in this paper. The constant phase, ϕ_c, represents the carrier phase shift observed at the receiver due to propagation delay and an arbitrary, but known, local reference time.

Additive channel noise, $n(t)$, can be thought of as an additional source of modulation that is random and unwanted. The composite signal, $s_c(t)$, in a narrowband IF filter is the sum of the noiseless signal and narrowband noise:

$$s_c(t) = r(t)\,A_c \cos[2\pi f_c t + \phi_c + \phi(t)] + n(t) \tag{8}$$

Since the channel noise is narrowband, it can be represented in quadrature form [6] as:

$$n(t) = n_c(t)\cos[2\pi f_c t + \phi_c] - n_s(t)\sin[2\pi f_c t + \phi_c] \tag{9}$$

The quadrature noise components, $n_c(t)$ and $n_s(t)$, are baseband, zero-mean, uncorrelated, Gaussian noise signals with a spectral shape that is the same as the IF filter.

The quadrature form of the composite IF signal is easily computed as the sum of the quadrature forms of the modulated carrier and the narrowband noise,

$$s_c(t) = \hat{a}(t) A_c \cos[2\pi f_c t + \phi_c] - \hat{b}(t) A_c \sin[2\pi f_c t + \phi_c] \tag{10}$$

where:

$$\hat{a}(t) = a(t) + n_c(t)/A_c = \hat{r}(t)\cos[\hat{\phi}(t)] \tag{11}$$

$$\hat{b}(t) = b(t) + n_s(t)/A_c = \hat{r}(t)\sin[\hat{\phi}(t)] \tag{12}$$

and the "hat" indicates that the function has been corrupted by noise. The relationships among all the noise-corrupted modulation functions are the same as noiseless ones given in (4) through (7).

It is important to note that the channel noise terms are always scaled by the amplitude of the carrier signal, i.e. the representation chosen for this model accounts for the signal-to-noise ratio in a "natural" way. The computation of the sideband signals requires the implementation of a Hilbert transform in the receiver.

4. THE DEMODULATION PROCESS

Demodulation is the process of recovering the noise-corrupted modulation functions from the received signal. It can also be thought of as carrier removal or "carrier stripping". To properly demodulate a signal, the carrier parameters (amplitude, frequency, and phase constant) must be known or estimated by the receiver. In other words, $\{\hat{a}(t), \hat{b}(t)\}$, $\{\hat{l}(t), \hat{u}(t)\}$, or $\{\hat{r}(t), \hat{\phi}(t)\}$ cannot, in general, be extracted from the carrier without a knowledge of the carrier parameters. The carrier amplitude governs the linearity and dynamic range requirements of the receiver as well as the design of the signal digitizer. Carrier frequency changes (not those due to frequency modulation) and/or frequency channelization require that the receiver be precisely tuned. Coherent modulation/demodulation schemes also require precise estimation (tracking) of both carrier frequency and carrier phase.

The noise-corrupted modulation functions will be called **demodulation functions** and A_c, f_c and ϕ_c will be called **carrier parameters**. To be effective, a receiver must produce good estimates of the carrier parameters and then successfully extract the demodulation functions. However, the original modulation functions can never be exactly recovered because they have been irretrievably corrupted by noise.

The most general approach to demodulation is to implement "I-Q" or quadrature detection and then remove the carrier by obtaining good estimates of the carrier parameters. Figure 1 shows a basic quadrature detector. To begin, the received signal is multiplied by the local-carrier quadrature functions as shown in the following equations:

$$x_1(t) = s_c(t) 2 A_0 \cos[\theta_0(t)] \tag{13}$$

$$y_1(t) = s_c(t) 2 A_0 \sin[\theta_0(t)] \tag{14}$$

where:

$$\theta_0(t) = 2\pi f_0 t + \phi_0 - \eta(t) \tag{15}$$

$$\frac{d}{dt}\theta_0(t) = 2\pi f_0 - \frac{d}{dt}\eta(t) \tag{16}$$

The parameters A_0, f_0, ϕ_0, and $\theta_0(t)$ for the local carrier, represent estimates of A_c, f_c, ϕ_c, and $\theta_c(t)$, respectively. The additive noise term, $\eta(t)$, is the error in the receiver's estimation of the carrier phase function.

When the trigonometric products representing the mixer outputs are expanded using trig identities, the equations representing the mixer outputs can be expressed as:

$$x_1(t) = \hat{r}(t) K_0 \cos[\theta_c(t) + \theta_0(t) + \hat{\phi}(t)] + \hat{r}(t) K_0 \cos[\theta_c(t) - \theta_0(t) + \hat{\phi}(t)] \tag{17}$$

and,

$$y_1(t) = \hat{r}(t) K_0 \sin[\theta_c(t) + \theta_0(t) + \hat{\phi}(t)] - \hat{r}(t) K_0 \sin[\theta_c(t) - \theta_0(t) + \hat{\phi}(t)] \tag{18}$$

Lowpass zonal filtering [7] (an "ideal" lowpass filter) can be applied to the multiplier outputs to reject the "sum" terms and pass the "difference" terms so that the filter outputs are,

$$x_2(t) = \hat{r}(t) K_0 \cos[\theta_c(t) - \theta_0(t) + \hat{\phi}(t)] = \hat{r}(t) K_0 \cos[\psi(t)] \tag{19}$$

$$y_2(t) = -\hat{r}(t) K_0 \sin[\theta_c(t) - \theta_0(t) + \hat{\phi}(t)] = -\hat{r}(t) K_0 \sin[\psi(t)] \tag{20}$$

where the substitution, $\psi(t) = \theta_c(t) - \theta_0(t) + \hat{\phi}(t)$, will be used to simplify the notation.

This widely-known result is valid only if the received signal is narrowband and the zonal filter passes an undistorted version of the entire spectrum of the difference term. In practical designs, the

zonal filter is implemented using a lowpass filter having a stopband attenuation that adequately suppresses the sum-frequency output while still passing the entire frequency spectrum of the difference-frequency output. If the lowpass filter distorts the difference-frequency spectrum, this result assuming zonal filtering is not valid. The technical literature frequently uses the symbols I and Q to represent these quadrature outputs. The quadrature outputs of the lowpass zonal filters contain all of the recoverable information about the transmitted signal as well as the undesired noise caused by both the channel and the imperfect estimation of the local-carrier phase function.

The quadrature detector outputs can be processed using summing, scaling, squaring, square root, inverse tangent, and Hilbert transform operations to estimate all of the modulation functions as follows:

$$r(t) \sim \hat{r}(t) = \frac{1}{K_0}\sqrt{x_2^2(t) + y_2^2(t)} \tag{21}$$

$$\phi(t) \sim \hat{\phi}(t) \sim \psi(t) = \arctan\left[\frac{-y_2(t)}{x_2(t)}\right] \tag{22}$$

$$a(t) \sim \hat{a}(t) \sim \frac{1}{K_0}\, x_2(t) \tag{23}$$

$$b(t) \sim \hat{b}(t) \sim \frac{-1}{K_0}\, y_2(t) \tag{24}$$

$$l(t) \sim \hat{l}(t) \sim \frac{1}{2}\left[x_2(t) - \tilde{y}_2(t)\right] \tag{25}$$

$$u(t) \sim \hat{u}(t) \sim \frac{1}{2}\left[x_2(t) + \tilde{y}_2(t)\right] \tag{26}$$

The estimates for the phase function, quadrature functions, and the upper and lower sideband signals are valid only if the receiver is "tracking" the carrier, i.e. only if $\theta_0(t) + \eta(t) = \theta_c(t)$. The double approximations are used to indicate the separate influences of channel noise and local oscillator noise in estimating all the outputs that are dependent on phase.

For specific types of modulation, there can be ways to recover the demodulation functions that

avoid the need for a quadrature detector. For example, a square-law-based envelope detector, represented by the following equation, can be used to recover the envelope function without the use of a quadrature detector:

$$x_4(t) = \sqrt{h_L(t) * \left[s_c^2(t)\right]} = \frac{1}{2}A_c\hat{r}(t) \tag{27}$$

For phase modulation, a simple phase-locked loop can be used. These two methods are represented by the block diagram of Figure 2. In this figure, x_5 is the output of the bandpass limiter which removes the envelope modulation and preserves the phase modulation. The PLL phase output is x_6 which is an estimate of the phase modulation function. The lower branch of the figure represents the processing needed for envelope demodulation.

In summary, a generalized demodulator must: 1) have a prior knowledge of, or estimate, the carrier parameters, 2) generate the estimated carrier for use in an I/Q detector (or a correlator), and 3) recover the demodulation functions.

5. GENERALIZED COSTAS DEMODULATOR

The Generalized Costas Demodulator (GCD) of Figure 3 represents a general method of signal demodulation. Its essential elements are, 1) a quadrature detector, 2) carrier estimation, and 3) a Hilbert transform. The signal processing blocks to the right of the quadrature detector implement Equations 21 through 26.

This general-purpose demodulator block diagram represents all of the governing equations necessary to demodulate any type of signal. It can be implemented using DSP if proper bandpass signal digitization is used. The next section will discuss various issues for this digital implementation.

6. DISCRETE-TIME DEMODULATION

Discrete-Time (or DSP) demodulation refers the concept of implementing the demodulation process in digital signal processing hardware and software [8] [9] [10] [11] [12]. A successful DSP demodulator design requires the proper use of 1) sampler prefiltering, 2) sampling design (including aliased sampling), 3) digital lowpass filtering, and 4) decimation. In addition, adequate algorithms must be available for the following operations: 1) multiplication, 2) squaring, 3) square root, and 4)

arctangent. Care must be taken to insure that the noise caused by number truncation in the algorithms does not degrade the signal-to-noise ratio in the demodulator.

The signal can be sampled 1) after the bandpass filter, 2) after the mixers, or 3) after the lowpass filters. A knowledge of the signal spectrum at each point in the block diagram is necessary if the design is to avoid signal aliasing. Also, this knowledge will help the designer to implement decimation where it will be the most effective.

Sampling after the Bandpass Filter. Baseband or aliased passband sampling [10] can be used after the bandpass filter as long as the sampling frequency, f_s, is chosen such that the spectrum of the "sum" frequency term can be adequately filtered while still passing the "difference" frequency term. This requirement is imposed because of the multiplication that is performed in the sine and cosine mixers. When using the governing equations, the carrier frequency, f_c, must be interpreted as the carrier frequency after aliased sampling. In actual designs, some spectral folding is acceptable as long as the lowpass filter can still effectively remove all of the spectrum associated with the "sum" frequency, i.e., it performs like a zonal filter. Decimation can be applied to the outputs of the lowpass filters to reduce the processing requirements for subsequent DSP operations. Care must be taken not to decimate to the point where subsequent operations might produce undesirable spectral folding.

Sampling after the Mixers. Baseband sampling can be used after the mixers as long as the folding frequency, $f_s/2$, is high enough to prevent the folding of the "sum" frequency terms into the equivalent passband of the lowpass filters. Some decimation can be applied to the outputs of the lowpass filters but the number rate should still be at least four times the cutoff frequency.

Sampling after the Lowpass Filters. If the carrier frequency is too high for adequate-quality aliased sampling, it is still possible to sample after the lowpass filters and then use DSP algorithms for the remaining demodulation processing. Because nonlinear operations such a squaring are used, the sampling rate (number rate) at the filter output must be much higher that the Nyquist rate. For example, Fourier analysis shows that the spectrum of x_2 or y_2 will be doubled after squaring so the number rate must be at least four times the cutoff frequency of the lowpass filter. Designers should carefully analyze the output spectrum for each specific design and choose the number rate accordingly.

To summarize, the signal spectrum at the sampling point and all subsequent numerical operations should be carefully studied before selecting the sampling rate and the amount of decimation to be applied. Many designs can be effective with aliased sampling of the bandpass filter output and a

number rate at the quadrature outputs of at least four times the lowpass filter cutoff frequency.

7. CARRIER ESTIMATION

The generation of a local replica for the carrier is required for proper operation of the Generalized Costas Demodulator or any synchronous demodulator. In order to generate a local replica, the receiver must be capable of estimating the carrier parameters A_c, f_c, and ϕ_c by using one or more of the signals represented by equations 10, 19, or 20. The required accuracy of the estimation for each parameter will depend upon the modulation type and the available signal-to-noise ratio.

Carrier estimation is accomplished in a variety of ways using a combination of filtering, phase tracking, and nonlinear operations such as squaring or bandpass limiting. The governing equations are summarized below.

Narrowband filtering. If a pilot carrier or other carrier frequency component is present and the modulation sidebands are sufficiently separated from the carrier, narrowband filtering can be used:

$$s_c(t) * h_n(t) \approx A_c \cos[2\pi f_c t + \phi_c] \tag{28}$$

In this case, the carrier phase constant also models the delay in the filter.

Bandpass limiter. When only envelope modulation is present, a bandpass limiter (hard limiter followed by a narrowband filter) can be used:

$$sgn\left\{s_c(t)\right\} * h_n(t) = \frac{4a}{\pi} \cos[2\pi f_c t + \phi_c] \tag{29}$$

The signum function, sgn{ }, [3] represents the action of the hard limiter. The symmetrical limiter clipping levels are $\pm a$.

Phase-locked Loop. For some types of modulation, phase-locked loop can be used to regenerate the carrier. For best performance, a PLL is often used in conjunction with hard limiters and/or filters. The PLL local oscillator produces a waveform with a fundamental frequency component of:

$$s_0(t) = A_0 \sin[2\pi f_0 t + \phi_0 - \eta(t)] \tag{30}$$

where $f_0 = f_c$ and $\phi_0 = \phi_c$ when the loop is properly tracking.

Costas Loop. The Costas loop is similar to the PLL in that it controls a local oscillator that tracks the carrier frequency and phase. The error signal for the loop is obtained by averaging the product of the quadrature outputs as represented below.

$$\begin{aligned}\left\langle x_2(t)\, y_2(t)\right\rangle &= -\hat{r}^2(t)\, K_0^2 \cos[\psi(t)]\sin[\psi(t)] \\ &= \frac{-K_0^2}{2}\left\langle \hat{r}^2(t)\sin[2\psi(t)]\right\rangle \\ &\approx -K_2\left\langle 2\psi(t)\right\rangle \approx 0\end{aligned} \tag{31}$$

The control loop works to keep the average value of $\psi(t)$ at zero.

Squaring Loop. The squaring loop [13] is used for binary phase-shift keyed (BPSK) signals and works by filtering or phase locking to the second harmonic of the squared narrowband signal represented below. Notice that by doubling the phase function, the phase modulation has been removed.

$$s_c^2(t) = \frac{\hat{r}^2(t)\, A_c^2}{2}\left[1 + \cos[2\theta_c(t) + 2\hat{\phi}(t)]\right] \tag{32}$$

The carrier is regenerated using a divide-by-two on the PLL oscillator output.

Multiplier, m-th order. The squaring loop idea can be applied to m-ary phase modulation by obtaining the m-th harmonic of the narrowband signal as represented by the equation:

$$\begin{aligned}s_c^m(t) &= \hat{r}^m(t)\cos^m[\theta_c(t) + \hat{\phi}(t)] \\ &= \hat{r}^m(t)\left[\cdots + k_m \cos[m\theta_c(t) + m\hat{\phi}(t)] + \cdots\right]\end{aligned} \tag{33}$$

Derivative. An estimate of the carrier frequency can be obtained by applying the derivative to the output of the bandpass limiter [14]. This result is:

$$\begin{aligned}\frac{d}{dt}\left[sgn\left\{s_c(t)\right\} * h_n(t)\right] &= -\frac{4a}{\pi}\sin[2\pi f_c t + \phi_c + \hat{\phi}(t)]\left[2\pi f_c + \frac{d}{dt}\hat{\phi}(t)\right] \\ &= -\left[8af_c + \frac{4a}{\pi}\frac{d}{dt}\hat{\phi}(t)\right]\sin[2\pi f_c t + \phi_c + \hat{\phi}(t)]\end{aligned} \tag{34}$$

The average value of the envelope function of the result is used for the estimate:

$$f_c \approx -\frac{1}{8a}\left\langle -[8af_c + \frac{4a}{\pi}\frac{d}{dt}\hat{\phi}(t)]\right\rangle \tag{35}$$

For this approach to be valid, the average value of the derivative of the phase modulation function

must be zero.

For some types of modulation it is not possible to obtain an estimate of the carrier parameters directly from the signal. Single-sideband modulation is an example of this. For these cases, the receiver must have an accurate prior knowledge of the carrier frequency.

8. CONCLUSIONS

The successful development of hardware and software designs for DSP demodulation requires a good understanding of the governing equations for demodulation and the aliasing problems associated with discrete-time nonlinear operations. Using the governing equations, it is possible to develop one design, the Generalized Costas Demodulator, that will work for any type of modulation. For new designs, aliased sampling at the output of the bandpass prefilter should be considered first. A number rate at least twice the Nyquist rate should be used at the output of the lowpass filters in the quadrature detector.

9. APPENDIX

An example will now be given to illustrate how the modulation functions are selected for a well-known modulation type, Minimum-Shift Keying (MSK). In MSK, one of four possible waveforms are sent using a phase-coherent frequency-shift keying technique. The four possible waveforms are generated by selecting one of two possible frequencies and one of two possible phases. The selections are restricted such that either the slope (MSK-1) or the magnitude (MSK-2) of the transmitted signal is zero at the time the next waveform is selected.

To insure these smooth transitions, the carrier frequencies are restricted to the values,

$$f_c = \frac{n + \frac{1}{2}}{2T_b} \tag{36}$$

where n is an integer (usually large) and T_b is the interval between data bits. The two frequencies transmitted by MSK are related to T_b and the carrier frequency as follows:

$$f_1 = \frac{n}{2T_b} = f_c - \frac{1}{4T_b} \tag{37}$$

$$f_2 = \frac{n+1}{2T_b} = f_c + \frac{1}{4T_b} \tag{38}$$

For MSK-1, the modulation functions defined for the time interval $kT_b \le t \le (k+1)\,T_b$ are

$$a_k(t) = I_k \cos\left[\frac{\pi t}{2T_b}\right] \tag{39}$$

$$b_k(t) = -Q_k \sin\left[\frac{\pi t}{2T_b}\right] \tag{40}$$

$$r_k(t) = 1 \tag{41}$$

$$\phi_k(t) = -\left[\frac{\pi t}{2T_b}\right] I_k Q_k + \frac{\pi}{2}[1 - Q_k] \tag{42}$$

where the data values, d_k, are used to create two, staggered, ½-rate sequences as follows:

1) for k even and $0 \le k \le N$

$$\begin{aligned} I_k &= I_{k+1} = d_k \\ Q_k &= d_{k-1} \end{aligned} \tag{43}$$

2) for k odd and $1 \le k \le N$

$$\begin{aligned} Q_k &= Q_{k+1} = d_k \\ I_k &= d_{k-1} \end{aligned} \tag{44}$$

Since the sequences are staggered, only one of the sequences can change its value at each data epoch, kT_b. The phase functions for odd and even k are,

$$\phi_k(t) = -\left[\frac{\pi t}{2T_b}\right] d_k d_{k-1} + \frac{\pi}{2}[1 - d_{k-1}] \quad \textit{for k even} \tag{45}$$

$$\phi_k(t) = -\left[\frac{\pi t}{2T_b}\right] d_k d_{k-1} + \frac{\pi}{2}[1 - d_k] \quad \textit{for k odd} \tag{46}$$

and the waveform for the k-th time interval is:

$$\begin{aligned} s_k(t) &= I_k \cos\left[\frac{\pi t}{2T_b}\right] A_c \cos\left[2\pi f_c t + \phi_c\right] \\ &\quad + Q_k \sin\left[\frac{\pi t}{2T_b}\right] A_c \sin\left[2\pi f_c t + \phi_c\right] \\ &= I_k A_c \cos\left[2\pi f_c t - \frac{\pi t}{2T_b} I_k Q_k + \phi_c\right] \\ &= A_c \cos\left[2\pi f_c t - \frac{\pi t}{2T_b} I_k Q_k + \frac{\pi}{2}[1 - I_k] + \phi_c\right] \end{aligned} \tag{47}$$

For the demodulation of MSK-1, the phase differences are used to estimate $I_k Q_k$.

$$\Delta\phi_k = \phi_k((k+1)T_b) - \phi_{k-1}(kT_b) = -\frac{\pi}{2} I_k Q_k \tag{48}$$

$$I_k Q_k = -\frac{2}{\pi}\Delta\phi_k \tag{49}$$

$$d_k = I_k Q_k d_{k-1} = -\frac{2}{\pi}\Delta\phi_k d_{k-1} \tag{50}$$

10. ACKNOWLEDGEMENTS

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12. FIGURES

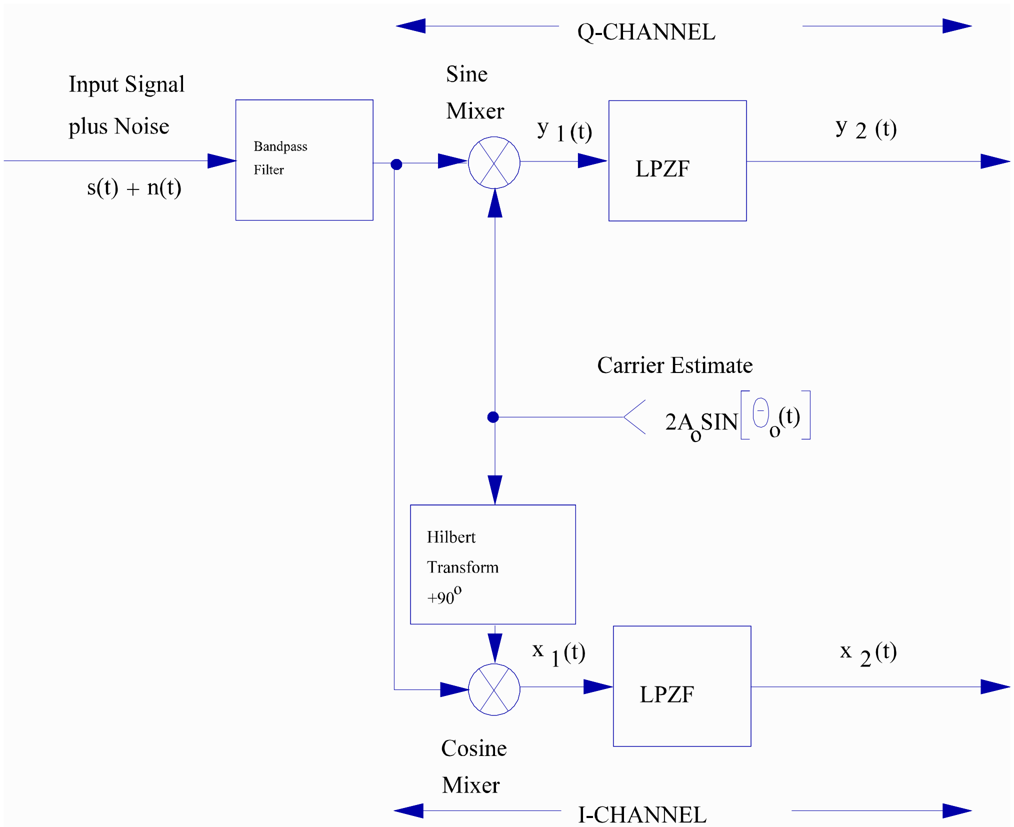
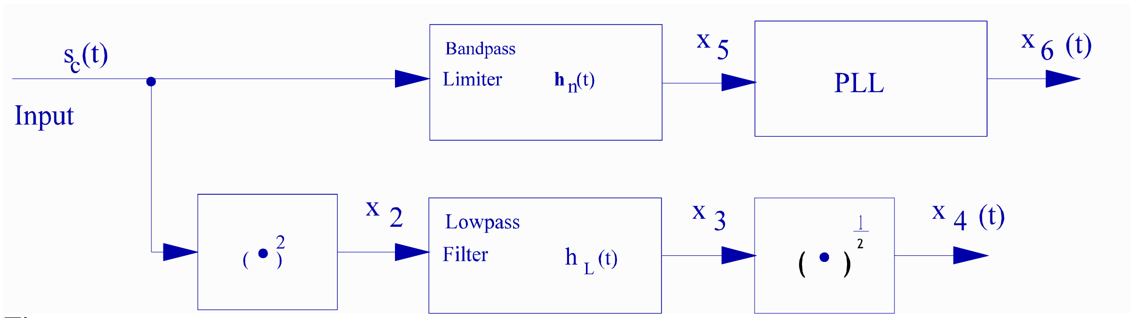


Figure 1 Basic Quadrature Detector



$s_c(t)$
Input
Bandpass
Limiter
$h_n(t)$
x_5
PLL
$x_6(t)$
$(\bullet)^2$
x_2
Lowpass
Filter
$h_L(t)$
x_3
$(\bullet)^{\frac{1}{2}}$
$x_4(t)$

Figure 2

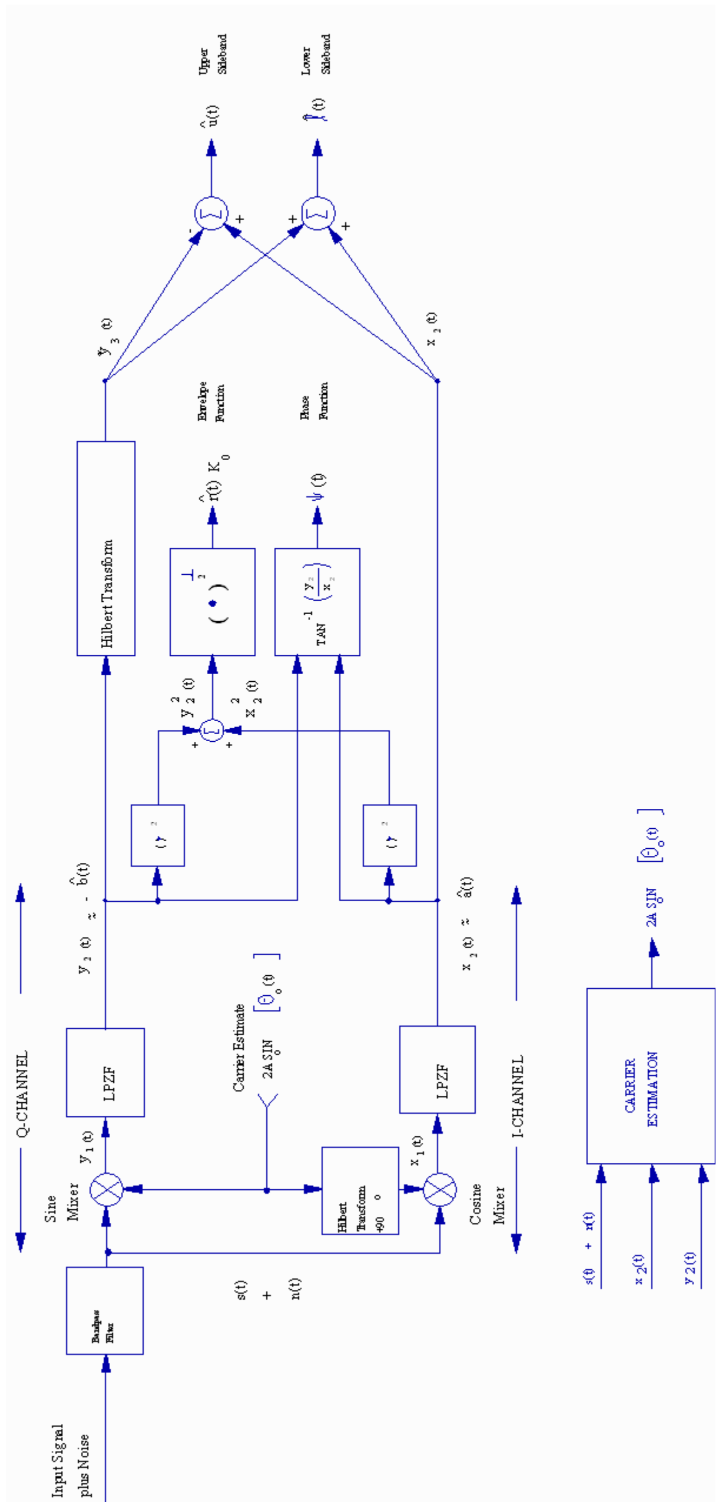


Figure 3 Generalized Costas Demodulator (GCD)